Investigation of Shock Configurations Induced by a Plume Impinging upon a Perpendicular Plate

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Abstract. The present work is an investigation of the normal impingement of an underexpanded jet upon a plane surface for different external pressures. Computations were carried out based on the quasigasdynamic equations and compared with the experimental results obtained in the SR3 low-density facility of the laboratoire d'Aérothermique.

1 Introduction

Jets issued from satellite control thrusters or from stage separation retro-rockets may impinge upon spacecraft walls, antennas or solar arrays, causing dynamic and thermal loads. In the particular case of normal impingement, those effects depend strongly upon the configuration of the shocks induced both by the surface and by the external atmosphere, if any. A numerical simulation of a jet impinging upon a perpendicular surface has been considered for the operating conditions of experiments that were carried out in the SR3 low-density facility.

Nitrogen plumes were issued from a 10° half-angle conical nozzle with stagnation temperatures $T_0 = 600$ K (variant A) and $1100$ K (variant B) and stagnation pressures $p_0 = 0.5$ bar (see Table 1). The background pressure $p_\infty$ was varied in the range from 8 to 330 Pa. Two flat plates were designed for pressure and heat transfer measurement, respectively and located at a distance $z_{max} = 40r_e = 0.1644m$ from the nozzle, were $r_e$ is the nozzle exit radius. It was found [1] that for low values of the background pressures $p_\infty = 8, 16, 33, Pa$, the shock induced ahead of the plate intersected the jet axis within the first expansion cell. For higher values of $p_\infty$ the central part of the plate was downstream of the plume first recompression wave.

2 Quasigasdynamic Equations

The numerical interpretation is based on the quasigasdynamic (QGD) system of equations. Theoretical investigation and numerical implementation of QGD system are summarized in [2]. The gasdynamic system consists of three differential
Table 1. Flow parameters

<table>
<thead>
<tr>
<th>Quantities</th>
<th>Notation</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nozzle exit pressure</td>
<td>( p_e ) (Pa)</td>
<td>1612</td>
<td>1654</td>
</tr>
<tr>
<td>Nozzle exit temperature</td>
<td>( T_e ) (K)</td>
<td>224.9</td>
<td>415.4</td>
</tr>
<tr>
<td>Stagnation temperature</td>
<td>( T_0 ) (K)</td>
<td>600</td>
<td>1100</td>
</tr>
<tr>
<td>Nozzle exit Mach number</td>
<td>( M_{ae} )</td>
<td>2.89</td>
<td>2.88</td>
</tr>
<tr>
<td>Nozzle wall temperature</td>
<td>( T_{wm} )</td>
<td>350</td>
<td>400</td>
</tr>
<tr>
<td>Knudsen number</td>
<td>( Kn = \lambda_e / 2 r_e )</td>
<td>2.89 \cdot 10^{-4}</td>
<td>6.03 \cdot 10^{-4}</td>
</tr>
</tbody>
</table>

Equations accounting for conservation of mass, momentum and total energy

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{J} = 0, \quad (1)
\]

\[
\frac{\partial (\rho u_i)}{\partial t} + \nabla \cdot (\rho u_i u_j - (2/3) g^{ik} \nabla_j u^k) = \nabla \cdot (\Pi^{ik}), \quad (2)
\]

\[
\frac{\partial E}{\partial t} + \nabla \cdot (E + p) + \nabla \cdot q^i = \nabla \cdot (\Pi^{ik} u^k). \quad (3)
\]

To close the system (1)-(3) the mass flux vector \( \mathbf{J} \), the shear-stress tensor \( \Pi^{ik} \), and the heat flux vector \( q^i \) must be expressed as a function of the macroscopic flow quantities: density \( \rho \), velocity components \( u_i \), and pressure \( p \). Different choices for \( \mathbf{J}, \Pi^{ik}, q^i \) lead either to the Navier-Stokes equations, or to the present QGD equations. Navier-Stokes equations are derived from

\[
J_i^{NS} = \rho u_i, \quad q_i^{NS} = -\kappa \nabla^2 T,
\]

\[
\Pi^{ik}_{NS} = \mu \left( \nabla^2 u_i + \nabla^2 u_k - (2/3) \eta^{ik} \nabla_j u^j \right) + \eta^{ik} \nabla_j u_j,
\]

Here the involved gasdynamic variables \( \rho, u_i \), and \( p \) are instantaneous space averaged quantities, and \( \Pi^{ik}_{NS} \) is the Navier–Stokes shear-stress tensor, \( \eta^{ik} \) is the metric tensor, \( \mu \) and \( \kappa \) are the viscosity and heat conductivity coefficients, respectively, \( \eta \) is the second viscosity coefficient (bulk viscosity).

In contrast to the Navier–Stokes equations, if the gasdynamic quantities \( \rho, u_i \), and \( p \) are defined by means of time-space averaging, instead of space averaging, the system (1)-(3) can be closed by other ways, in particular by

\[
J_i = \rho u_i - \tau (\nabla_j (\rho u_i u_j) + \nabla_j p), \quad q_i^{NS} = -\tau \rho \nabla^2 u_i (u_j \nabla_j s), \quad (4)
\]

\[
\Pi^{ik} = \Pi^{ik}_{NS} + \tau u_i (\rho u_j \nabla_j u_k + \nabla^k p) + \tau g^{ik} (u_j \nabla_j p + \gamma p \nabla_j u_j), \quad (5)
\]

where \( \varepsilon = p / \rho (\gamma - 1) \) is the specific internal energy, \( s \) is the specific entropy, \( \tau = \mu / \rho \) is the Maxwell relaxation time. Equations (1)-(3) with (4)-(5) form the QGD equations, a system where the mass, momentum, and total energy conservation laws, and the entropy theorem are valid as for the classic Navier–Stokes
system. QGD equations were obtained also by a kinetical approach that consists in integrating a model kinetic equation multiplied by collisional invariants.

For slightly nonequilibrium flows the time-space averaged quantities and the space-averaged quantities are similar, QGD and Navier–Stokes systems differing by $O(\tau)$. For stationary flows, the dissipative terms (terms in $\tau$) in the QGD equations appear as Navier–Stokes terms. QGD and Navier–Stokes equations differing by additional terms whose asymptotic order is $\tau^2$ for $\tau \to 0$, or, in the dimensionless form of the equations, $O(Kn^2)$ for $Kn \to 0$. The boundary layer approximation for QGD equations leads to the classic Prandtl equation system.

3 Problem formulation and computational results

The problem under consideration is solved in $(r - z)$ formulation using QGD equations. The viscosity law is taken as $\mu \sim T^\omega$. The computational domain is shown in Fig. 1. As boundary conditions, no-slip was assumed for the left wall and for the plate (with temperature $T_w = 293$ K), usual symmetry conditions and “soft” conditions were applied on the axis and on the upper boundary, respectively. At nozzle exit, a laminar boundary layer profile was prescribed [3].

For a solution of QGD equations the upwind-type splitting scheme was used based on the additional dissipative terms of QGD system (see [4] for details). Second-order accuracy was obtained by applying the standard MUSCL approximation and “minmod” limiter. For the diffusive terms a second-order central difference scheme was applied.

Computations were carried out for nitrogen ($\gamma = 1.4, \omega = 0.74, Pr = 0.736$), for 2 variants of flow conditions at nozzle exit (jets A and B) and for 6 values of the background pressure $p_{\infty} = 8, 16, 33, 66, 160$ and 330 Pa, thus reproducing the experimental conditions (Table 2). In the present paper only the variants for jet B are discussed. The results for jet A were found to be similar.

For all variants, rectangular computational grids were used with steps $h_{\text{rmin}} = 0.1 r_e$ and $h_z = 0.25 r_e$ (variants 1 and 2), or $h_z = 0.5 r_e$ (variants 3–6). The number of steps varied from $161 \times 115$ ($r_{\text{max}} = 100 r_e$) for variant 1 to $81 \times 73$ ($r_{\text{max}} = 40 r_e$) for variant 6. For a solution of the QGD equations an upwind-type splitting scheme was used based on the additional dissipative terms of QGD system (see [4] for details). Second-order accuracy was obtained by applying the standard MUSCL approximation and “minmod” limiter. For the diffusive terms a second-order central difference scheme was applied.
In Table 2 is shown the extent ız of the first expansion cell as found in numerical simulations of jet B when it interacts with the plate, together with the estimations of this quantity for a free jet, based on formulas by Abramovich (ız\textsuperscript{AB}), Ashkenas (ız\textsuperscript{AS}) and Lengrand (ız\textsuperscript{L}). The distance from the nozzle exit to the plate is equal to ız\textsubscript{max}/r\textsubscript{e} = 40. When the estimated extent of the first expansion cell is larger then the distance to the plate (variants 1 and 2), the flow is stationary. It was found that the steady-state solution was reached after a number of oscillations. The level of the oscillations increased with the background pressure. Here a normal shock is formed, much like a Mach reflection in a free jet: the local Mach number exceeds unity before the shock and it is less than unity behind the shock. The corresponding T and p distributions for variant 2 are shown in Figs. 2 and 3. For variant 3, the first expansion cell in the undisturbed jet extends close to the plate position. For this variant the same structure as for variant 2 is found, but the numerical solution is nonstationary: the shock wave position oscillates in the limits shown in Table 2. Time-dependent pressure oscillations for the point located at r = 0 are presented in Fig. 6. Two

<table>
<thead>
<tr>
<th>variant</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>p\textsubscript{\infty} (Pa)</td>
<td>8</td>
<td>16</td>
<td>33</td>
<td>66</td>
<td>160</td>
<td>330</td>
</tr>
<tr>
<td>ız\textsuperscript{AB}/r\textsubscript{e}</td>
<td>87.5</td>
<td>61.9</td>
<td>43.1</td>
<td>30.5</td>
<td>19.6</td>
<td>13.6</td>
</tr>
<tr>
<td>ız\textsuperscript{AS}/r\textsubscript{e}</td>
<td>105.9</td>
<td>74.9</td>
<td>52.2</td>
<td>36.9</td>
<td>23.7</td>
<td>16.5</td>
</tr>
<tr>
<td>ız\textsuperscript{L}/r\textsubscript{e}</td>
<td>80.8</td>
<td>56.0</td>
<td>40.0</td>
<td>28.0</td>
<td>18.0</td>
<td>12.6</td>
</tr>
<tr>
<td>ız\textsubscript{c}/r\textsubscript{e}</td>
<td>33.5</td>
<td>29.5</td>
<td>28.5 ~ 30.0</td>
<td>32.0</td>
<td>20.5</td>
<td>15.0</td>
</tr>
<tr>
<td>reflection</td>
<td>MR</td>
<td>MR</td>
<td>MR</td>
<td>RR</td>
<td>RR</td>
<td>RR</td>
</tr>
</tbody>
</table>
examples of pressure $p(r)/p_e$ and heat transfer rate $q(r)$ distributions on the plate are shown in Figs. 4 and 5 for times $t_1$ and $t_2$, that correspond to the smallest $t_1$ and largest $t_2$ abscissas of the shock wave.

For variants 4, 5 and 6, the distance to the plate is larger than the extent of the first expansion cell. For variant 4, one expansion cell is formed ahead the plate, and for variant 6 two expansion cells are formed there. In these variants the flowfield remains stationary, but reflections now have a regular character (RR in Table 2); the local Mach numbers exceeds unity ahead and behind the shock wave. Isobars and isotherms for variants 4 are shown in Figs. 8 and 9. Mach number distributions along $z$ for variants 2 (MR) and 4 (RR) are shown in Fig. 7.

The computed distributions of non-dimensionalized wall pressure $p/p_e$ and heat transfer rate $q$ for variants 1, 2, 4, 6 are shown in Figs. 10-13. Comparison with experimental data from [1] in the central point of the plate is given in Table

![Fig. 4. Wall pressure, variant 3](image1)

![Fig. 5. Wall heat transfer, variant 3](image2)

![Fig. 6. Time dependent wall pressure, variant 3](image3)

![Fig. 7. Mach number distributions, variants 2 and 4](image4)
3. Pressure profiles exhibit the same qualitative evolution as described in [1], but the calculated results appear to be slightly lower than experimental values.

<table>
<thead>
<tr>
<th>variant</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_{exp} )</td>
<td>0.038</td>
<td>0.048</td>
<td>0.068</td>
<td>0.7</td>
<td>1.2</td>
<td>1.9</td>
</tr>
<tr>
<td>( p_{cal} )</td>
<td>0.029</td>
<td>0.038</td>
<td>0.039</td>
<td>0.071</td>
<td>0.49</td>
<td>0.47</td>
</tr>
<tr>
<td>( q_{exp} )</td>
<td>( 6 \cdot 10^3 )</td>
<td>( 1.2 \cdot 10^3 )</td>
<td>( 9.5 \cdot 10^2 )</td>
<td>( 4.5 \cdot 10^4 )</td>
<td>( 1.2 \cdot 10^7 )</td>
<td>( 8 \cdot 10^4 )</td>
</tr>
<tr>
<td>( q_{cal} )</td>
<td>( 3.1 \cdot 10^3 )</td>
<td>( 3.0 \cdot 10^3 )</td>
<td>(-95 )</td>
<td>(+250 )</td>
<td>( 5.6 \cdot 10^7 )</td>
<td>( 2.5 \cdot 10^7 )</td>
</tr>
</tbody>
</table>

**Fig. 8.** Isotherms, variant 4

**Fig. 9.** Isobars, variant 4

**Fig. 10.** Wall pressure, variants 1, 2

**Fig. 11.** Wall pressure, variants 4, 6
4 Conclusion

Numerical results exhibit a good global agreement with experiment, and provide information on those quantities not measured in the experiment (for example, nonstationary regimes as studied in [5]).

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References