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MATHEMATICAL PHYSICS

Artificial Dissipation Coefficients in Regularized Equations of Supersonic Aerodynamics

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Abstract—A method for introducing artificial dissipation coefficients into a numerical algorithm based on the quasi-gasdynamic system of equations is proposed. The method applies to aerodynamic flows with large Mach and Reynolds numbers. Simulation results for the supersonic flow over the X-43 aircraft are presented as an illustration. The pressure distribution over the aircraft surface is obtained, which can be used to calculate the aerodynamic characteristics of X-43.

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The system of quasi-gasdynamic (QGD) equations was constructed in [1–3] as a regularized form of the Navier–Stokes (NS) equations. A numerical algorithm based on this system (known as the QGD algorithm) and related kinetically consistent difference schemes have been successfully applied to gas flow simulation and exhibited efficiency when implemented on modern high-performance computer systems. With the use of the QGD algorithm, a wide variety of gas flows ranging from moderately rarefied ones to high-velocity dense gas flows can be simulated in a unified manner. The capabilities of the algorithm as applied to rarefied, subsonic, or supersonic flows are determined by the choice of dissipation coefficients involved in these models. In this paper, we describe available variants of these coefficients and propose a method for introducing artificial dissipation coefficients for simulating aerodynamic flows with large Mach and Reynolds numbers.

The QGD system can be treated as the NS equations averaged over a short spatiotemporal interval, which leads to smoothing, or regularization, of the original system of equations. Due to this smoothing, additional dissipative terms proportional to a small parameter τ having the dimension of time appear in the equations. These terms have the form of second derivatives with respect to spatial coordinates and are nonlinear functions of the flow parameters.

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Following [2, 3], the QGD system of equations in Cartesian coordinates without external forces or heat sources can be represented in the form

$$\frac{\partial}{\partial t}\rho + \nabla_i j^i_m = 0, \tag{1}$$

$$\frac{\partial}{\partial t}\rho u^j + \nabla_i(j^i_m u^j) + \nabla^j p = \nabla_i \Pi^{ij}, \tag{2}$$

$$\frac{\partial}{\partial t}E + \nabla_i(j^i_m H) + \nabla_i q^i = \nabla_i(\Pi^{ij} u_j). \tag{3}$$

Here, traditional notation is used: ρ is the gas density, u^i are the components of the macroscopic velocity of the gas, and p is the pressure. The total energy per unit volume E and the total specific enthalpy H of an ideal polytropic gas with the ratio of specific heats γ are given by the formulas

$$E = \frac{\rho u^2}{2} + \frac{p}{\gamma - 1}, \quad H = \frac{E + p}{\rho}.$$

The mass flux density j^i_m is defined as

$$j^i_m = \rho(u^i - w^i), \quad w^i = \frac{\tau}{\rho}(\nabla_j \rho u^i u^j + \nabla^i p).$$

The viscous stress tensor Π^{ij} and the heat flux q^i are written as

$$\Pi^{ij} = \Pi^{ij}_{NS} + \tau u_i \rho\left(u_k \nabla^k u_j + \frac{1}{\rho}\nabla_j p\right) + \tau\delta^{ij}(u_k \nabla^k p + \gamma p \nabla^k u_k), \tag{4}$$

$$\Pi^{ij}_{NS} = \mu\left(\nabla^i u^j + \nabla^j u^i - \frac{2}{3}\nabla^k u_k\right) + \zeta\delta^{ij}\nabla^k u_k, \tag{5}$$

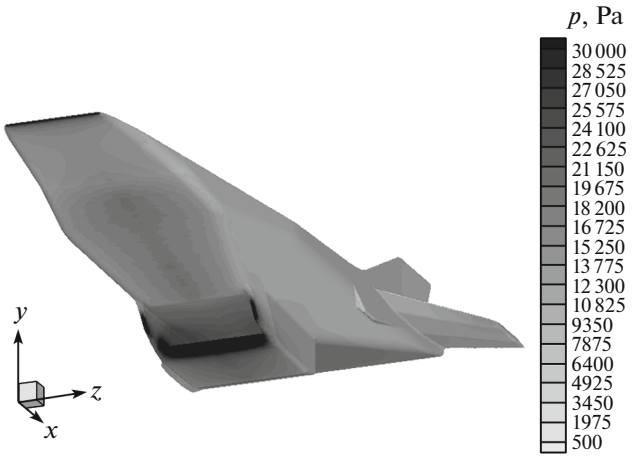

Fig. 1. General view of the aircraft and level lines of pressure on its surface.

$$q^i = q^i_{NS} - \tau u^i \rho\left(u_j \nabla^j \varepsilon + p u_j \nabla^j \frac{1}{\rho}\right), \qquad q^i_{NS} = -\kappa \nabla^i T. \tag{6}$$

Here, $\varepsilon = \dfrac{p}{\rho(\gamma - 1)}$ is the internal energy per unit mass of the gas, Π^{ij}_{NS} and q^i_{NS} are the viscous stress tensor and the heat flux in the NS equations; μ, ζ, and κ are the shear and bulk viscosity coefficients and the thermal conductivity, respectively; and T is the temperature of the gas.

The shear viscosity coefficient μ can be determined as a temperature dependence:

$$\mu = \mu_0 \left(\frac{T}{T_0}\right)^{\omega}, \tag{7}$$

where μ_0 is the viscosity of the gas at the temperature T_0 and $0.5 \le \omega \le 2$ is the exponent in the intermolecular interaction law [4].

The bulk viscosity can be calculated using an approximating formula based on kinetic theory [2, 5]:

$$\varsigma = \mu\left(\frac{5}{3} - \gamma\right), \tag{8}$$

while the thermal conductivity is given by

$$\kappa = \frac{\mu}{\mathrm{Pr}(\gamma - 1)}. \tag{9}$$

For a viscous polytropic gas, the coefficient τ determining additional dissipation in the QGD algorithm has the order of the characteristic time between collisions of the gas particles. Its value is related to the shear viscosity coefficient and can be calculated as [2, 3]

$$\tau = \frac{\mu}{p\mathrm{Sc}}, \tag{10}$$

where Sc is the Schmidt number, which is close to 1 for gases [4].

This definition of the dissipative coefficients corresponds to the derivation of the QGD equations from kinetic theory [1] and is adequate for the numerical simulation of rarefied gas flows by applying the QGD algorithm. This algorithm is based on the finite volume method [6] combined with the Euler method for approximating the difference equations and with second-order accurate central differences for computing all flux variables, including the convective terms, on control volume faces [2, 3]. The conditional stability of the numerical algorithm is ensured by regularizing τ-terms.

However, the above-described dissipation is insufficient for ensuring the stability of the QGD algorithm in the simulation of transonic dense gas flows. To enhance the stability of the numerical algorithm in this case, the dissipative coefficient τ (10) can be modified by adding a term depending on the spatial mesh size and flow parameters, namely,

$$\tau = \frac{\mu}{p\mathrm{Sc}} + \frac{\alpha h}{c}, \tag{11}$$

where h is the characteristic size of a spatial cell, c is the local speed of sound, and α is a tuning parameter, which is usually set to a constant of order 1 (see, e.g., [1–3, 7]).

In a boundary layer, the terms with parameter τ are close to zero, since the QGD equations in the boundary layer approximation degenerate into the Prandtl equations [3]. Thus, artificial dissipation introduced into the QGD algorithm as described above does not degrade the accuracy of simulation of the near-wall flow or viscous fluxes toward the wall, while smoothing out the high gradients of the solution in internal flow regions.

However, at high supersonic velocities, the introduced τ-viscosity is again insufficient. Numerical computations have shown that, in this case, the numerical algorithm can be stabilized by adding an artificial term to the bulk viscosity coefficient (8), namely,

$$\varsigma = \mu\left(\frac{5}{3} - \gamma\right) + \delta \frac{h}{c} p. \tag{12}$$

The value of the regularizing term is also determined by the local parameters and the tuning coefficient δ.

Note that the terms with coefficient ς in the viscous stress tensor (5) are on the main diagonal, so the positive term added to ς in (12) provides enhanced stability of the difference scheme. On the other hand, the interaction of the flow with the wall is determined by off-diagonal elements of the viscous stress tensor, and the difference analogues of the diagonal tensor components turn out to be relatively small near the wall.

Thus, the above-described artificial dissipation introduced into the algorithm allows us to smooth out

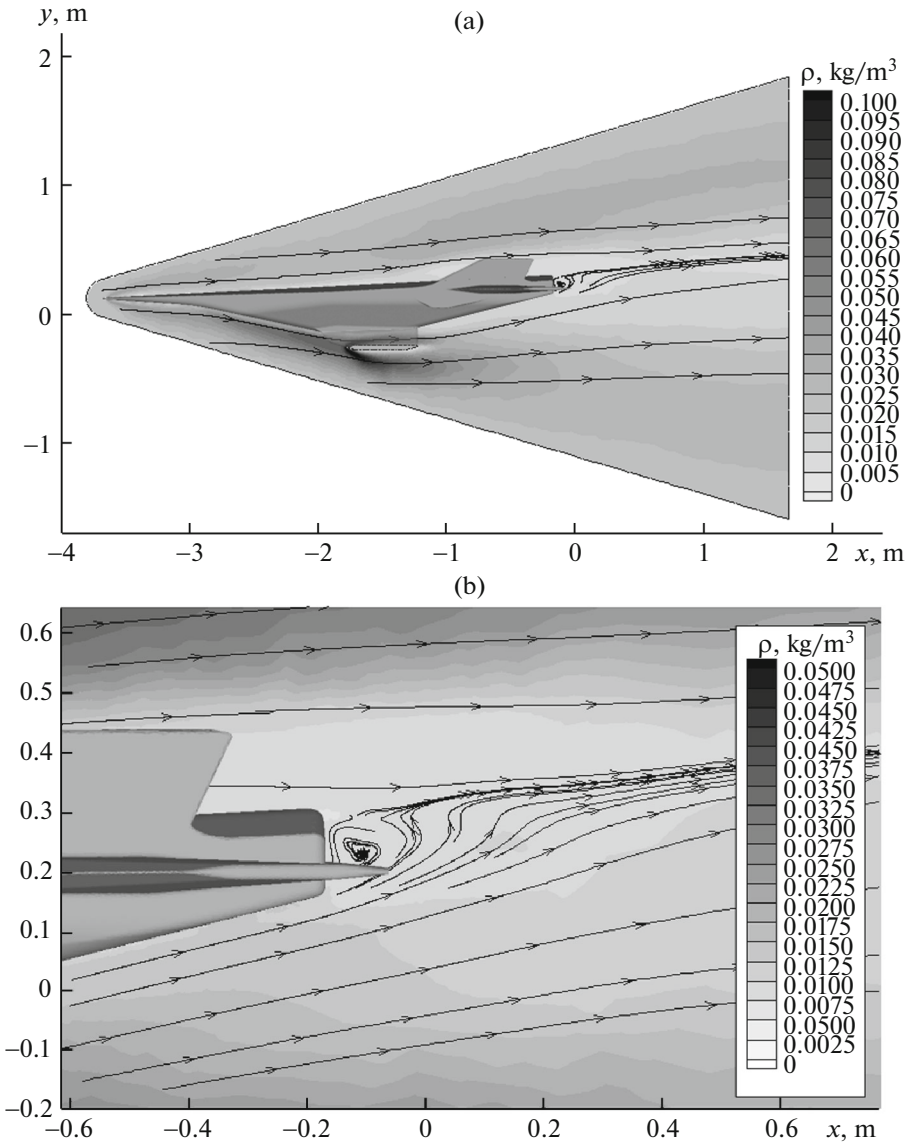

Fig. 2. Level lines of density and streamlines in the computational domain around the X-43 aircraft.

the numerical solution near shock waves, while preserving the structure of the shear viscosity near the solid surfaces bounding the flow, which is important for simulating near-wall effects.

To illustrate the application of the above-described procedure, we present the numerical results obtained for the supersonic flow around the X-43 aircraft [8]. The QGD algorithm was constructed for system (1)–(9) with dissipation coefficients (7) and (9) and artificial additions (11) and (12). Here, artificial dissipation of form (12) was used for the first time for essentially supersonic flows.

The numerical simulation was based on a modified computer code [9] intended for unsteady viscous gas flows for bodies of arbitrary shape on tetrahedral unstructured meshes. The computations were performed on the multiprocessor computing system K-100 [10] at the Keldysh Institute of Applied Mathematics of the Russian Academy of Sciences. To save CPU time, the numerical algorithm was parallelized as based on a decomposition of the computational domain and the use of the MPI standard. The computer code exhibited good scalability and fairly high efficiency of parallelization. The computations were performed using 128 processor cores.

The flow parameters were specified as corresponding to actual flights of X-43 [8]; namely, we used the Mach number M = 7, the flight height $H = 29$ km, and the Reynolds number (per 1 m) $\mathrm{Re} = 3.1 \times 10^6$ 1/m. The angle of attack was 2°. Additionally, we specified the gas constant $R = 287$ J/(kg K), the ratio of specific

heats $\gamma = \frac{7}{5}$, the Prandtl number $\mathrm{Pr} = \frac{14}{19}$, and the intermolecular interaction exponent $\omega = 0.74$. The numerical tuning coefficients α and δ in (11) and (12) were determined by the required accuracy and stability of the algorithm and were set to 1 and 15, respectively.

The general view of X-43 is presented in Fig. 1, together with level lines of pressure on its surface. Pressure maxima can be seen at the leading edge of the aircraft and at the air intake inlet. The pressure distribution makes it possible to calculate aerodynamic characteristics of the aircraft, such as drag and lift coefficients.

Figure 2 shows level lines of density and streamlines in the plane of symmetry of X-43. Figure 2a presents the general view of the aircraft. On the lower surface of the forebody, we can see the formation of a shock wave interacting with the air intake and the formation of supersonic flow inside it. An unsteady vortex flow is formed behind the aircraft. This zone is shown on a larger scale in Fig. 2b.

The capabilities of the QGD algorithm as applied to the simulation of unsteady vortex flows, including the laminar–turbulent transition, were demonstrated in [7, 11, 12] for the decay of the Taylor–Green vortex in free space and for the near-wall turbulent flow in the Couette problem. To expand the possibilities of its practical use, the QGD algorithm was incorporated as an additional core into the open software OpenFOAM [13].

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